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Quantum effects on electron-proton bremsstrahlung spectrum in a two-component plasma

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Abstract. The electron-proton low energy bremsstrahlung process is investigated in a two-component plasma. The corrected Kelbg potential taking into account the quantum effects is applied to describe the electron-proton interaction potential in a two-component plasma. The straight-line trajectory method is applied to the motion of the projectile electron in order to investigate the variation of the bremsstrahlung cross-section as a function of the scaled impact parameter, thermal de Broglie wavelength, projectile energy, and photon energy. The results show that the quantum-mechanical effects decrease the bremsstrahlung cross-sections when the de Broglie wavelength (λ) is greater than the Bohr radius (a_0). It is also found that the quantum effects are important only for the region of impact parameters $b < 3a_0$.

PACS. 52.20.-j Elementary processes in plasmas

1 Introduction

The bremsstrahlung process [1–12] in plasmas has received much attention since this process has been widely used for plasma diagnostics in laboratory and astrophysical plasmas. Recently, in weakly coupled plasmas [6, 10], the low energy bremsstrahlung process has been investigated using the Yukawa type Debye-Hückel potential [12] with the classical trajectory method. The Debye-Hückel potential describes the properties of a low density plasma and corresponds to a pair correlation approximation. The plasmas described by the Debye-Hückel model are called ideal plasmas since the average energy of interaction between particles is small compared to the average kinetic energy of a particle [13]. However, in the region of partial degeneration and strong coupling region, the interaction potential is different from a pure Coulomb or screened Coulomb potential because of the strong coupling and quantum effects of nonideal particle interaction. Such systems can be observed in laboratory and astrophysical plasmas. Then, the bremsstrahlung spectrum due to electron-ion Coulomb scattering in partially degenerated strong coupling quasi-classical plasmas is different from that in classical ideal plasmas. Thus, in this paper we investigate the bremsstrahlung process in the electron-proton scattering in a two-component quasi-classical plasma. An effective potential model so called the corrected Kelbg potential [14,15] including classical as well as quantum-mechanical effects such as the Heisenberg principle and the Pauli exclusion principle is applied to describe the interaction potential between the projectile electron and target ion in a two-component quasi-classical plasma. The straight-line trajectory method [2, 3,16] is applied to obtain the differential bremsstrahlung cross-section as a function of the scaled impact parameter, thermal de Broglie wavelength, projectile energy, and photon energy.

In Section 2, we discuss the expression of the bremsstrahlung cross-section in scattering of the low energy projectile electron with the target proton in a two-component-plasma described by the correct Kelbg potential. In Section 3 we obtain the closed form of the differential radiation cross-section using the components of the force parallel ($F_{\|}$) and perpendicular ($F_{\perp}$) to the velocity of the projectile electron. We also investigate the variation of the radiation cross-section with a change of the impact parameter and the thermal de Broglie wavelength. Finally, in Section 4, discussions are given.

2 Classical bremsstrahlung cross-section

The expression of the low energy bremsstrahlung cross-section [3] is given by

$$\mathrm{d}\sigma_{\mathrm{b}} = 2\pi \int \mathrm{d}b\, b\, \mathrm{d}w_{\omega}(b), \tag{1}$$

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where b is the impact parameter and $\mathrm{d}w_\omega$ is the differential probability of emitting a photon of frequency ω within $\mathrm{d}\omega$ when a projectile particle changes its velocity in collisions with a static target system. For all impact parameters, $\mathrm{d}w_\omega$ is given by the Larmor formula [17] for the emission spectrum of a nonrelativistic accelerated electron:

$$\mathrm{d}w_\omega = \frac{8\pi e^2}{3\hbar m^2 c^3} |\mathbf{F}_\omega|^2 \frac{\mathrm{d}\omega}{\omega}, \quad (2)$$

where m is the electron rest mass and $\mathbf{F}_\omega$ is the Fourier coefficient of the force $\mathbf{F}(t)$ acting on the projectile electron

$$\mathbf{F}_\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t}\, \mathbf{F}(t). \quad (3)$$

The straight-line character of the motion of the projectile electron in the field of the scattering potential can also be investigated using the modified hyperbolic-orbit curved trajectory method [18]. It has been known that the bremsstrahlung cross-section using the straight-line trajectory method and the curved trajectory method are almost identical for impact parameters $b > a_0$, where a_0 $(= \hbar^2/me^2)$ is the Bohr radius [9]. Thus, the straight-line trajectory method for the region of impact parameters $b > a_0$ is quite reliable. Since the investigation of the quantum-mechanical effects on the electron-proton bremsstrahlung process in a two-component quasi-classical plasma for $b > a_0$ is the main purpose of this work, the straight-line trajectory method has been retained throughout this paper. In the straight-line trajectory method, the position of the projectile electron can be represented as $\mathbf{r}(t) = \mathbf{b} + \mathbf{v}t$ with $\mathbf{b} \cdot \mathbf{v} = 0$, where v is the velocity of the projectile electron. We can set up coordinate axes to compute $\mathbf{F}_\omega$ so that the electron orbit is in the collision plane; then

$$|\mathbf{F}_\omega|^2 = \left|F_{\|\omega}\right|^2 + |F_{\perp\omega}|^2, \quad (4)$$

where the Fourier coefficients $F_{\|\omega}$ and $F_{\perp\omega}$ are the components of force parallel and perpendicular to the projectile velocity, respectively.

In the region of partial degenerate and strong coupling, the interaction of charged particles cannot be represented by a pure Coulomb potential but it can be introduced by effective pair potentials [19–21]. The Kelbg potential [19] obtained by the first-order perturbation theory is known to be a good approximation for two-particle Slater sums in the case of small parameter ξ $(= e^2/\lambda k_\mathrm{B}T)$ for large separation of the electron-proton interaction, where λ $(= \hbar/\sqrt{2mk_\mathrm{B}T})$ is the thermal de Broglie wavelength, k_B denotes the Boltzmann constant, and T is the plasma temperature. However, the Kelbg potential has a deviation from the exact value of the Slater sums for small separations. Very recently, the corrected Kelbg potential [14,15] was obtained using the Slater sum and its first derivative for small separations and for low temperatures $k_\mathrm{B}T < 0.3Ry$, where Ry $(= me^4/2\hbar^2 \cong 13.6$ eV) is the Rydberg constant. The corrected Kelbg potential [15] for the electron-proton interaction in a two-component plasma including all quantum effects (the Heisenberg uncertainty principle and the Pauli exclusion principle) can be shown to be

$$V(r) = \frac{e^2}{r}\left\{1 - \mathrm{e}^{-r^2/\lambda^2} + \sqrt{\pi}(r/\lambda)\left[1 - \mathrm{erf}(r/\lambda)\right]\right\} - k_\mathrm{B}T\tilde{A}(\xi)\mathrm{e}^{-r^2/\lambda^2}, \quad (5)$$

where $\mathrm{erf}(x)$ $[= (2/\sqrt{\pi}) \int_0^x \mathrm{d}t\, \mathrm{e}^{-t^2}]$ is the error function and the temperature-dependent parameter $\tilde{A}(\xi)$ is represented by

$$\tilde{A}(\xi) = -\sqrt{\pi}\xi + \ln\left[\sqrt{\pi}\xi^3\left(\zeta(3) + \frac{1}{4}\zeta(5)\xi^2\right) + 4\sqrt{\pi}\xi \int_0^\infty \mathrm{d}y \frac{y\, \mathrm{e}^{-y^2}}{1 - \mathrm{e}^{-\pi\xi/y}}\right], \quad (6)$$

here $\zeta(p)$ is the Riemann zeta functions [22]. In the absence of the quantum effects $(\lambda \to 0)$, the corrected Kelbg potential $V(r)$ goes over into the pure Coulomb potential $V_C(r) = e^2/r$. The use of the corrected Kelbg potential (Eq. (5)) and the straight-line trajectory impact parameter approach allows us to derive analytic formulas for the Fourier coefficients of the force after some algebra:

$$F_{\mu\omega} = -\frac{e^2}{\pi v a_0} \bar{F}_{\mu\omega}(\bar{b}, \bar{\lambda}, \bar{\varepsilon}, \bar{E}), \quad (\mu = \|,\ \perp), \quad (7)$$

where the scaled Fourier coefficients $\bar{F}_{\|\omega}$ and $\bar{F}_{\perp\omega}$ are given by

$$\bar{F}_{\|\omega} = \mathrm{i}\int_0^\infty \mathrm{d}\tau\, \tau \sin\eta\tau \left[\frac{1}{(\bar{b}^2 + \tau^2)^{3/2}}\left(1 - \mathrm{e}^{-(\bar{b}^2+\tau^2)/\bar{\lambda}^2}\right) + \frac{\tilde{A}(2\bar{\lambda})}{\bar{\lambda}^4}\mathrm{e}^{-(\bar{b}^2+\tau^2)/\bar{\lambda}^2}\right], \quad (8)$$

$$\bar{F}_{\perp\omega} = \int_0^\infty \mathrm{d}\tau\, \bar{b} \cos\eta\tau \left[\frac{1}{(\bar{b}^2 + \tau^2)^{3/2}}\left(1 - \mathrm{e}^{-(\bar{b}^2+\tau^2)/\bar{\lambda}^2}\right) + \frac{\tilde{A}(2\bar{\lambda})}{\bar{\lambda}^4}\mathrm{e}^{-(\bar{b}^2+\tau^2)/\bar{\lambda}^2}\right], \quad (9)$$

here τ $(\equiv vt/a_0)$ is the scaled time, $\eta \equiv \omega a_0/v$, $\bar{b}$ $(\equiv b/a_0)$ is the scaled impact parameter, $\bar{\lambda}$ $(\equiv \lambda/a_0)$ is the scaled de Broglie wavelength, and $\xi = 2\bar{\lambda}$. Then, in nonrelativistic limits, the classical differential bremsstrahlung cross-section is found to be

$$\mathrm{d}\sigma_\mathrm{b} = \frac{16}{3}\frac{\alpha^3 a_0^2}{\bar{E}}\frac{\mathrm{d}\omega}{\omega}\int_{\bar{b}_\mathrm{min}}^\infty \mathrm{d}\bar{b}\, \bar{b}\left(\left|\bar{F}_{\|\omega}\right|^2 + \left|\bar{F}_{\perp\omega}\right|^2\right), \quad (10)$$

where α $(= e^2/\hbar c \cong 1/137)$ is the fine structure constant and $\bar{E}$ $(\equiv E/Ry = mv^2/2Ry)$ is the scaled kinetic energy of the projectile electron. Here the scaled minimum impact parameter $\bar{b}_\mathrm{min}$ corresponds to the closest distance of approach at which the electrostatic potential energy of interaction is equal to the maximum possible energy transfer [23], *i.e.*, $mv^2/2 \approx V(b)$.

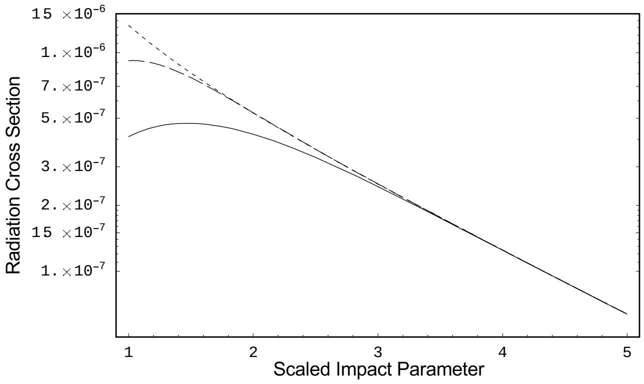

Fig. 1. The scaled doubly differential radiation cross-section ($\mathrm{d}^2\chi_\mathrm{b}/\mathrm{d}\bar{\varepsilon}\mathrm{d}\bar{b}$) (Eq. (13)) in units of πa_0^2 as a function of the scaled impact parameter ($\bar{b} = b/a_0$) for the hard photon radiation case ($\varepsilon/E = 0.9$) when $\bar{E} = 0.5$. The solid line represents the radiation cross-section for $\bar{\lambda}$ $(= \lambda/a_0) = 1.5$. The dashed line represents the radiation cross-section for $\bar{\lambda} = 1$. The dotted line represents the radiation cross-section for $\bar{\lambda} = 0.5$.

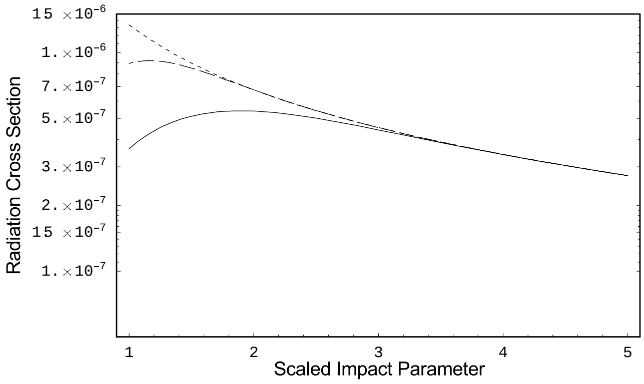

Fig. 2. The scaled doubly differential radiation cross-section ($\mathrm{d}^2\chi_\mathrm{b}/\mathrm{d}\bar{\varepsilon}\mathrm{d}\bar{b}$) (Eq. (13)) in units of πa_0^2 as a function of the scaled impact parameter ($\bar{b} = b/a_0$) for the soft photon radiation case ($\varepsilon/E = 0.1$) when $\bar{E} = 0.5$. The solid line represents the radiation cross-section for $\bar{\lambda}$ $(= \lambda/a_0) = 1.5$. The dashed line represents the radiation cross-section for $\bar{\lambda} = 1$. The dotted line represents the radiation cross-section for $\bar{\lambda} = 0.5$.

3 Radiation cross-section

The differential radiation cross-section for the bremsstrahlung process is defined by [17]

$$\frac{\mathrm{d}\chi_\mathrm{b}}{\mathrm{d}\varepsilon} \equiv \frac{\mathrm{d}\sigma_\mathrm{b}}{\hbar \mathrm{d}\omega}\hbar\omega, \tag{11}$$

$$= \frac{16}{3}\frac{\alpha^3 a_0^2}{\bar{E}}\int_{\bar{b}_{\min}}^{\infty} \mathrm{d}\bar{b}\,\bar{b}\left(\left|\bar{F}_{\parallel\omega}\right|^2 + \left|\bar{F}_{\perp\omega}\right|^2\right), \tag{12}$$

where ε $(\equiv \hbar\omega)$ is the radiation photon energy. In nonrelativistic limits, the parameter η can be rewritten as $\eta = \bar{\varepsilon}/2\sqrt{\bar{E}}$, where $\bar{\varepsilon}$ $(\equiv \hbar\omega/Ry)$ is the scaled photon energy. Then, the scaled doubly differential radiation cross-section in units of πa_0^2 for the electron-proton bremsstrahlung process, *i.e.*, the scaled differential radiation cross-section per scaled impact parameter, can be presented as

$$\frac{\mathrm{d}^2\chi_\mathrm{b}}{\mathrm{d}\bar{\varepsilon}\mathrm{d}\bar{b}}/\pi a_0^2 = \frac{16}{3\pi}\frac{\alpha^3}{\bar{E}}\bar{b} \times \left[\left|\bar{F}_{\parallel\omega}(\bar{b},\bar{\lambda},\bar{\varepsilon},\bar{E})\right|^2 + \left|\bar{F}_{\perp\omega}(\bar{b},\bar{\lambda},\bar{\varepsilon},\bar{E})\right|^2\right] \cdot \tag{13}$$

In order to investigate the quantum-mechanical effects on the scaled doubly differential bremsstrahlung radiation, we consider three cases of the thermal de Broglie wavelength: $\bar{\lambda}$ $(\equiv \lambda/a_0) = 0.5$, 1, and 1.5, and we consider two cases for the ratio of the radiation photon energy to the kinetic energy of the projectile electron ε/E $(= 2\hbar\omega/mv^2) =$ 0.1 (soft photon radiation) and 0.9 (hard photon radiation). Here, we choose $\bar{E} = 0.5$ since the bremsstrahlung cross-section equation (1) is known to be reliable for low energy projectiles ($v < \alpha c$) [3]. Figure 1 shows the scaled doubly differential radiation cross-section ($\mathrm{d}^2\chi_\mathrm{b}/\mathrm{d}\bar{\varepsilon}\mathrm{d}\bar{b}$) in units of πa_0^2 as a function of the scaled impact parameter $\bar{b}$ $(= b/a_0)$ for the electron-proton bremsstrahlung process in a two-component plasma for the hard photon radiation case ($\varepsilon/E = 0.9$). Figure 2 shows the scaled doubly differential radiation cross-section as a function of the scaled impact parameter for the soft photon radiation case ($\varepsilon/E = 0.1$). As we can see in these figures, the quantum-mechanical effects decrease the radiation cross-sections for $\bar{\lambda} > 1$, *i.e.*, when the de Broglie wavelength (λ) is greater than the Bohr radius (a_0). It is found that the quantum-mechanical effects are important for the region of impact parameters $b < 3a_0$. For large impact parameters, the quantum-mechanical effects are almost negligible. Hence, the quantum effects are found to be only important near the target nucleus.

4 Discussions

We have investigated the quantum effects on the electron-proton bremsstrahlung process in a two-component plasma. The corrected Kelbg potential taking into account the quantum-mechanical effects is applied to describe the electron-proton interactions in a two-component plasma. The straight-line trajectory method is applied to the motion of the projectile electron in order to investigate the variation of the differential bremsstrahlung radiation cross-section as a function of the scaled impact parameter, thermal de Broglie wavelength, projectile energy, and photon energy. It is found that the quantum-mechanical effects substantially decrease the bremsstrahlung cross-sections when the de Broglie wavelength is greater than the Bohr radius. The quantum-mechanical effects are also found to be important for $b < 3a_0$. However, the quantum-mechanical effects on the bremsstrahlung cross-section are decreased with increasing the impact parameter. These results provide useful information on the electron-ion bremsstrahlung processes in quasi-classical two-component plasmas.

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